LESSON 3.4b

Analyzing the Discriminant

Today you will:

- Analyze the ***discriminant*** to determine the number and type of solutions
- Practice being a math translator

Core Vocabulary:

- Discriminant, p. 124 $b^2 - 4ac$

    …the stuff ***inside*** the square root…

- Tells us the number and type of solutions to the quadratic equation.

The Discriminant

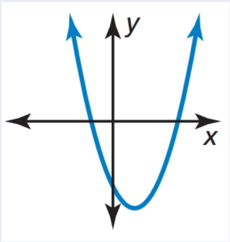
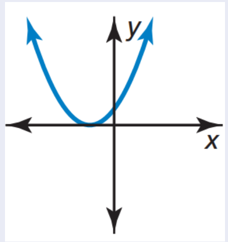
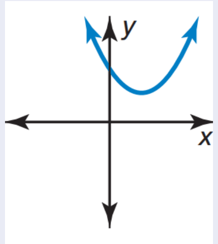
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the ***Discriminant***

The Discriminant tells us the number and type of solutions

Number: two, one or none

Type: real or imaginary

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercepts

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

a. $x^2 - 6x + 10 = 0$ **b.** $x^2 - 6x + 9 = 0$ **c.** $x^2 - 6x + 8 = 0$

SOLUTION

Equation	**Discriminant**	**Solution(s)**
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
a. $x^2 - 6x + 10 = 0$	$(-6)^2 - 4(1)(10) = -4$	Two imaginary: $3 \pm i$
b. $x^2 - 6x + 9 = 0$	$(-6)^2 - 4(1)(9) = 0$	One real: 3
c. $x^2 - 6x + 8 = 0$	$(-6)^2 - 4(1)(8) = 4$	Two real: 2, 4

Find a possible pair of integer values for a and c so that the equation $ax^2 - 4x + c = 0$ has one real solution. Then write the equation.

SOLUTION

In order for the equation to have one real solution, the discriminant must equal 0.

$b^2 - 4ac = 0$	Write the discriminant.
$(-4)^2 - 4ac = 0$	Substitute -4 for b.
$16 - 4ac = 0$	Evaluate the power.
$-4ac = -16$	Subtract 16 from each side.
$ac = 4$	Divide each side by -4.

Because $ac = 4$, choose two integers whose product is 4, such as $a = 1$ and $c = 4$.

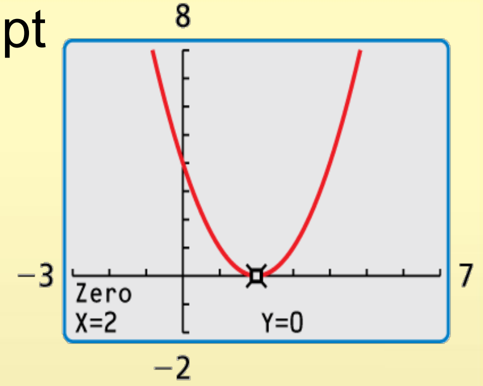
ANOTHER WAY

Another possible equation in Example 5 is $4x^2 - 4x + 1 = 0$. You can obtain this equation by letting $a = 4$ and $c = 1$.

So, one possible equation is $x^2 - 4x + 4 = 0$.

Check Graph $y = x^2 - 4x + 4$. The only x-intercept is 2. You can also check by factoring.

$$x^2 - 4x + 4 = 0$$
$$(x - 2)^2 = 0$$
$$x = 2 \checkmark$$



Solving Real-Life Problems

The function $h = -16t^2 + h_0$ is used to model the height of a *dropped object*.

h is the height (in feet) at time t (in seconds).

The starting height is h_0.

If the object is *launched or thrown*, an extra term $v_0 t$ is added. The initial vertical velocity is v_0 (in feet/sec).

$h = -16t^2 + h_0$ *dropped object*

$h = -16t^2 + v_0 t + h_0$ *launched or thrown object*

Affect of v_0:

If $v_0 < 0$ then object is falling/angling down

If $v_0 > 0$ then object is climbing/angling up

If $v_0 = 0$ then object is falling straight down

A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

SOLUTION

Because the ball is *thrown*, use the model $h = -16t^2 + v_0t + h_0$. To find how long the ball is in the air, solve for t when $h = 3$.

$h = -16t^2 + v_0t + h_0$ — Write the height model.

$3 = -16t^2 + 30t + 4$ — Substitute 3 for h, 30 for v_0, and 4 for h_0.

$0 = -16t^2 + 30t + 1$ — Write in standard form.

This equation is not factorable, and completing the square would result in fractions. So, use the Quadratic Formula to solve the equation.

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(1)}}{2(-16)}$$ $a = -16$, $b = 30$, $c = 1$

$$t = \frac{-30 \pm \sqrt{964}}{-32}$$ Simplify.

$t \approx -0.033$ or $t \approx 1.9$ — Use a calculator.

Reject the negative solution, −0.033, because the ball's time in the air cannot be negative. So, the ball is in the air for about 1.9 seconds.

Homework

Pg 127 #19-59 odd, 69